A dynamic monetary multi-sectoral model of production

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Though Keynes entitled his magnum opus The general theory of employment, interest and money (Keynes 1936), he acknowledged that money did not feature heavily in his technical analysis, and that he saw a substantial continuity between monetary analysis and the Marshallian model of supply and demand:

whilst it is found that money enters into the economic scheme in an essential and peculiar manner, technical monetary detail falls into the background. A monetary economy, we shall find, is essentially one in which changing views about the future are capable of influencing the quantity of employment and not merely its direction. But our method of analyzing the economic behavior of the present under the influence of changing ideas about the future is one which depends on the interaction of supply and demand, and is in this way linked up with our fundamental theory of value. We are thus led to a more general theory, which includes the classical theory with which we are familiar, as a special case. (Keynes 1936, p. xxii)

After Keynes, macroeconomics fragmented around the importance of both uncertainty—implicit in the statement above that “changing views about the future are capable of influencing the quantity of employment”, but strongly explicit elsewhere (Keynes 1936; Keynes 1937)—and money. Both concepts disappeared from mainstream macroeconomic analysis, to be replaced initially by IS-LM analysis—in which an exogenously determined money played a minor role, but uncertainty disappeared (Hicks 1937; Minsky 1975; Hicks 1981)—and ultimately by Real Business Cycle modeling (Kydland and Prescott 1982), in which “rational expectations” neutered uncertainty completely (Lucas 1972), and money was entirely absent.

On the periphery of the profession, a rump of self-described “Post Keynesians” clung to the position that both money and uncertainty were essential aspects of macroeconomics. Going far further than Keynes himself, this rump incorporated Schumpeter's arguments on the essential role of endogenously created money in financing growth (Schumpeter 1927; Schumpeter 1934; Moore 1979) and Fisher's debt-deflation perspective (Fisher 1933) to develop the “Financial Instability Hypothesis” (Minsky 1975; Minsky 1977; Minsky 1982; Minsky 1993), while it also rejected Marshallian analysis—following on this issue Sraffa (Sraffa 1926; Robertson, Sraffa et al. 1930) rather than Keynes. Others added insights from theoretical developments like complexity theory, which post-dated Keynes, to argue that the macro-economy was inherently cyclical (Goodwin 1967; Goodwin 1986; Goodwin 1990).

This rump was ignored by the mainstream, which over time expunged not only uncertainty and money but even Keynes himself from macroeconomics (despite the fact that the dominant segment of the mainstream described its work as “New Keynesian”). Mainstream macroeconomics became applied neoclassical microeconomics, as Oliver Blanchard, founding
editor of the journal *AER: Macro*, outlined in his survey of macroeconomics in 2009.

The most visible outcomes of this new approach are the dynamic stochastic general equilibrium (DSGE) models. They are models derived from micro foundations—that is, utility maximization by consumers-workers; value maximization by firms; rational expectations; and a full specification of imperfections, from nominal rigidities to some of the imperfections discussed above—and typically estimated by Bayesian methods. (Blanchard 2009, p. 223)

As the end of the first decade of the 21st century approached, the mainstream was triumphal. At the policy level, it took the credit for the decline in economic volatility since the early 1980s:

As it turned out, the low-inflation era of the past two decades has seen not only significant improvements in economic growth and productivity but also a marked reduction in economic volatility, both in the United States and abroad, a phenomenon that has been dubbed "the Great Moderation." Recessions have become less frequent and milder, and quarter-to-quarter volatility in output and employment has declined significantly as well. The sources of the Great Moderation remain somewhat controversial, but as I have argued elsewhere, *there is evidence for the view that improved control of inflation has contributed in important measure to this welcome change in the economy.* (Bernanke 2004; emphasis added)

At the level of pure theory, a similar contentment prevailed. Though he acknowledged one notable dissenter (Solow 2008), Blanchard's survey was unequivocal:

The state of macro is good. (Blanchard 2009, p. 210)

Few more poorly timed statements have ever been made by prominent economists. This paper was first published online as a working paper in August 2008 (Blanchard 2008)—one year after the event that is now regarded as the beginning of the financial crisis (New York Times 2007) and 8 months after the NBER's date for the commencement of the Great Recession (NBER 2011). Its publication as a journal paper in May 2009 preceded the NBER's date for the end of this recession by one month (a decision that I expect will prove premature).

Blanchard was forced into recanting his optimism less than a year later (Blanchard, Dell'Ariccia et al. 2010). But while he criticized macroeconomic policy prior to the crisis, he remained a believer in neoclassical theory itself:

Identifying the flaws of existing policy is (relatively) easy. Defining a new macroeconomic policy framework is much harder... *It is important to start by stating the obvious, namely, that the baby should not be thrown out with the bathwater. Most of the elements of the pre-crisis consensus, including the major conclusions from macroeconomic theory, still hold.* Among them, the ultimate targets remain output and inflation stability. The natural rate hypothesis holds, at least to a good enough approximation, and policymakers should not design policy on the assumption that there is a long-term trade-off between inflation and unemployment. Stable inflation must remain one of the major goals of monetary policy. Fiscal sustainability is of the essence, not only for the long term but also in
affecting expectations in the short term. (Blanchard, Dell'Ariccia et al. 2010, p. 207; emphasis added)

Blanchard's unwillingness to countenance the possibility that the Great Recession may be a Kuhnian critical anomaly for neoclassical macroeconomics (Bezemer 2011) is representative of this school of thought:

Indeed, the extreme severity of this great recession makes it tempting to argue that new theories are required to fully explain it... But ... it would be premature to abandon more familiar models just yet. (Ireland 2011, p. 1; emphasis added)

As a representative of the Post Keynesian and complexity theory rump, and one of the handful of economists to foresee the Great Recession (Keen 1995; Keen 2000; Keen 2006; Keen 2007; Keen 2007; Bezemer 2009; Bezemer 2011), I could not disagree more with Blanchard and his colleagues. Though neoclassical economists believe they are being methodologically sound in applying microeconomic concepts to model the macro-economy, deep research long ago established that this is a fallacy. The Sonnenschein-Mantel-Debreu conditions alone establish that even the microeconomics of demand in a single market cannot be derived by extrapolation from the behavior of a single utility-maximizing agent, let alone the macroeconomics of the whole economy. As Solow himself noted in the paper cited in Blanchard (2009, p. 210):

Suppose you wanted to defend the use of the Ramsey model as the basis for a descriptive macroeconomics. What could you say? ...

You could claim that ... there is no other tractable way to meet the claims of economic theory. I think this claim is a delusion. We know from the Sonnenschein-Mantel-Debreu theorems that the only universal empirical aggregative implications of general equilibrium theory are that excess demand functions should be continuous and homogeneous of degree zero in prices, and should satisfy Walras' Law. Anyone is free to impose further restrictions on a macro model, but they have to be justified for their own sweet sake, not as being required by the principles of economic theory. Many varieties of macro models can be constructed that satisfy those basic requirements without imposing anything as extreme and prejudicial as a representative agent in a favorable environment. (Solow 2008, p. 244; emphasis added; see also Solow 2001 and 2003)

I cover the myriad flaws in neoclassical macroeconomics in much more detail in Keen 2011b; suffice it to say here that, far from it being unwise to “throw the baby out with the bathwater”, neoclassical macroeconomics should never have been conceived in the first place. The Great Recession will hopefully prove to be the Biblical economic flood needed to finally sink this superficially appealing but fundamentally flawed vision of how the macro-economy functions.

How do I fault thee? Let me count the ways

The flaws of neoclassical macroeconomics are almost too numerous to enumerate, but the key weaknesses are:
1. Treating a complex monetary market economy as a barter system;

2. Assuming that the macro-economy is either in equilibrium (partial or general, perfect or imperfect), or that it will return to equilibrium rapidly if disturbed;

3. Modeling the entire economy using “applied microeconomics” and ignoring social class, when the Sonnenschein-Mantel-Debreu conditions (Sonnenschein 1972; Sonnenschein 1973; Kirman 1989; Shafer and Sonnenschein 1993) establish that, as Kirman put it:

   “we may well be forced to theorise in terms of groups who have collectively coherent behaviour. Thus demand and expenditure functions if they are to be set against reality must be defined at some reasonably high level of aggregation. The idea that we should start at the level of the isolated individual is one which we may well have to abandon” (Kirman 1992, p. 138);

4. Obliterating uncertainty from macroeconomic theory with the absurd proposition that a rational individual is someone who can accurately foresee the future—which is what “rational expectations” really means;¹

5. Persisting with a simplistic “money multiplier” model of money creation when the empirical evidence against this model is overwhelming (Holmes 1969; Moore 1979; Moore 1988; Kydland and Prescott 1990); and

6. Ignoring the pivotal roles of credit and debt in the macro-economy.

All these flaws are absent from the non-neoclassical rump—especially in the work of Minsky. But what the rump lacks, in comparison to the neoclassical mainstream, is a coherent mathematical expression of its model that is widely accepted within that school. In this paper I contribute to the development of such a model (though I appreciate that my model is a long way from being accepted by my peers) using a modeling framework—which I call Monetary Circuit Theory (MCT)—that, in contrast to the neoclassical litany of sins above:

1. Treats the economy as inherently monetary;

2. Makes no assumptions about the nature of equilibrium and models the economy dynamically;

3. Models behavior at the level of social classes rather than isolated agents;

4. Presumes rational but not prophetic behavior: people in social classes act in what they perceive as their best interests given information available, but do not attempt to forecast the future state of the economy (and they cannot do so in any case, because of the well-known features of complex systems);

5. Models the endogenous creation of money by the banking sector in a pure credit economy (later extensions will incorporate fiat money creation by governments); and

¹ Lucas stated as much in the paper in which he introduced “rational expectations” into macroeconomics, by stating that rational expectations was identical to assuming that future expectations were correct: “the hypothesis of adaptive expectations was rejected as a component of the natural rate hypothesis on the grounds that, under some policy [the gap between expected future inflation and actual future inflation] is non-zero. As the impossibility of a non-zero value for Expression 6 is taken as an essential feature of the natural rate theory, one is led simply to adding the assumption that Expression 6 is zero as an additional axiom, or to assume that expectations are rational in the sense of Muth.” (Lucas 1972, p. 54)
6. Gives credit and debt the pivotal roles in economic theory that the Great Recession has shown they have in the real world.

**A framework for monetary macroeconomics**

At one level, \( MCT \) is deceptively simple: all demand in the macroeconomy is treated as originating in bank accounts, where, in accordance with the empirical literature (Holmes 1969; Moore 1979, 1988; Kydland and Prescott 1990), the banking system has the capacity to endogenously create new credit-based money. The development of the framework is described elsewhere (see Keen 2006b, 2008, 2009); here I will simply illustrate \( MCT \) with the financial flows used in the model of the 19th century “free banking” system in Keen (2010).\(^2\) The core of \( MCT \) is a tabular layout of the financial relations between the economic entities in the model, where each column represents an aggregate bank account, and each row represents operations on and between those accounts.\(^3\)

**Table 1: Sample Financial Flows Godley Table**

<table>
<thead>
<tr>
<th>Account Name</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bank Vault</td>
<td>Firm Loan</td>
<td>Firm Deposit</td>
</tr>
<tr>
<td><strong>Symbol</strong></td>
<td>( B_V(t) )</td>
<td>( F_L(t) )</td>
<td>( F_D(t) )</td>
</tr>
<tr>
<td><strong>Initial conditions</strong></td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lend Money</td>
<td>-A</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Record Loan</td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound Debt</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service Debt</td>
<td>-B</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Record Payment</td>
<td>-B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposit Interest</td>
<td>C</td>
<td>-C</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>-D</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Deposit Interest</td>
<td>E</td>
<td>-E</td>
<td></td>
</tr>
<tr>
<td>Consume</td>
<td>F+G</td>
<td>-F</td>
<td>-G</td>
</tr>
<tr>
<td>Repay Loan</td>
<td>H</td>
<td>-H</td>
<td></td>
</tr>
</tbody>
</table>

\(^2\) The table differs slightly from that in the paper, since the columns have been re-ordered and renamed in accordance with standard accounting practice.

\(^3\) The table is similar to the Social Accounting Matrix approach of Wynne Godley (see Godley & Lavoie 2007a and 2007b), but has several differences that are explained in Keen 2009, pp. 162-167—notably that row operations do not have to sum to zero, and the economy is modeled in continuous rather than discrete time.
Using a symbolic algebra program, the placeholders A to H are then replaced by suitable functions:

\[
\begin{align*}
A & \rightarrow \frac{B_v(t)}{\tau_v(t)} \\
B & \rightarrow r_L(t) \cdot F_L(t) \\
C & \rightarrow r_D(t) \cdot F_D(t) \\
D & \rightarrow \frac{F_D(t)}{\tau_F(t)} \\
E & \rightarrow r_D(t) \cdot W_D(t) \\
F & \rightarrow \frac{W_D(t)}{\tau_w(t)} \\
G & \rightarrow \frac{B_v(t)}{\tau_g(t)} \\
H & \rightarrow \frac{F_L(t)}{\tau_F(t)} \\
\end{align*}
\]

The program then automatically derives a set of differential equations for this system, which can be analyzed symbolically or simulated numerically:

\[
\begin{align*}
\frac{d}{dt} B_v(t) &= \frac{F_L(t)}{\tau_L(t)} - \frac{B_v(t)}{\tau_v(t)} \\
\frac{d}{dt} F_L(t) &= \frac{B_v(t)}{\tau_v(t)} - \frac{F_L(t)}{\tau_L(t)} \\
\frac{d}{dt} F_D(t) &= \frac{B_v(t)}{\tau_v(t)} - r_L(t) \cdot F_L(t) + r_D(t) \cdot F_D(t) - \frac{F_D(t)}{\tau_F(t)} + \frac{W_D(t)}{\tau_w(t)} + \frac{B_E(t)}{\tau_B(t)} - \frac{F_L(t)}{\tau_L(t)} \\
\frac{d}{dt} W_D(t) &= \frac{F_D(t)}{\tau_F(t)} + r_D(t) \cdot W_D(t) - \frac{W_D(t)}{\tau_w(t)} \\
\frac{d}{dt} B_E(t) &= r_L(t) \cdot F_L(t) - r_D(t) \cdot F_D(t) - r_D(t) \cdot W_D(t) - \frac{B_E(t)}{\tau_B(t)} \\
\end{align*}
\]

4 This system has been implemented in the commercial programs Mathcad (www.ptc.com/mathcad), Mathematica (www.wolfram.com) and Matlab (www.mathlab.com), and a prototype of a standalone monetary simulation system called QED—for “Quesnay Economic Dynamics”—is freely downloadable from my website http://www.debtdeflation.com/blogs/qed/.

5 I explain the functions used in the exposition of the multisectoral model below.
This covers the financial side of the economy. The real economy is coupled to this via a price mechanism (and links between the wages flow—which determines employment—and investment, which is not shown in the simple model in Table 1, but which determines the capital stock in a larger model).

The price mechanism is derived analytically in Keen 2010 (pp. 17-18), and corresponds to the extensive empirical literature into how firms actually set prices—which has nothing to do with marginal cost and marginal revenue (see Lee 1998, Blinder et al. 1998, and Keen & Standish 2006 and 2010) but instead represents a markup on the wage costs of production

\[
\frac{dP}{dt} = -\frac{1}{\tau_p} \left( P - \frac{1}{1 - \sigma} \cdot \frac{W}{a} \right)
\]

The real economy itself is modeled using Goodwin’s growth cycle (Goodwin 1967; see also Blatt 1983, pp. 204-216), but expressed in absolute values (Employment, Wages, etc.) rather than ratios (rate of employment, wages share of output) as in Goodwin’s original model.

**Applying the framework: a “corn economy” with a financial crisis**

The sample Godley Table shown in Table 1 has to be extended to allow for investment, which as Schumpeter argued is the sound basis on which the credit system endogenously creates new debt-based money (Schumpeter 1934, pp. 95-101).

**Table 2: Godley Table for Corn Economy Model**

<table>
<thead>
<tr>
<th>Account Name</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bank Vault</td>
<td>Firm Loan</td>
<td>Worker Deposit</td>
</tr>
<tr>
<td><strong>Symbol</strong></td>
<td>FV(t)</td>
<td>FL(t)</td>
<td>FD(t)</td>
</tr>
<tr>
<td>Lend from Vault</td>
<td>-A</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Record Loan</td>
<td></td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Compound Debt</td>
<td></td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Service Debt</td>
<td></td>
<td>-C</td>
<td>C</td>
</tr>
<tr>
<td>Record Payment</td>
<td></td>
<td>-C</td>
<td></td>
</tr>
<tr>
<td>Debt-financed Investment</td>
<td></td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Record Investment Loan</td>
<td></td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td></td>
<td>-E</td>
<td>E</td>
</tr>
<tr>
<td>Deposit Interest</td>
<td></td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

\[6 \ \tau_p \text{ is the time constant in price setting, } \sigma \text{ is the share of income going to capitalists, and } a \text{ is labor productivity.} \]
This Godley Table results in the following generic system of financial flows:

\[
\begin{align*}
\frac{d}{dt} B_v(t) &= J - A \\
\frac{d}{dt} F_L(t) &= A - J + B - C + D \\
\frac{d}{dt} F_D(t) &= A - J - C + D - E + F + H + I \\
\frac{d}{dt} W_D(t) &= E + G - H \\
\frac{d}{dt} B_E(t) &= C - F - G - I
\end{align*}
\]

The substitutions for this table are show in Equation (1.5); the rates of lending, investment and loan repayment (respectively A, D and J in Table 2) are now functions of the rate of profit, and wage payments (E) are now wages times employment.

<table>
<thead>
<tr>
<th>Consumption</th>
<th>H+I</th>
<th>-H</th>
<th>-I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repay Loan</td>
<td>J</td>
<td>-J</td>
<td></td>
</tr>
<tr>
<td>Record Repayment</td>
<td>-J</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The basic causal cycle in the Goodwin model (to which the financial flows above are attached) is quite simple. Causation flows from left to right in equations (1.6) to (1.14):
• The level of the physical capital stock $K_r$ determines the level of physical output $Y_r$ per year:

$$\frac{K_r(t)}{v} = Y_r(t)$$  \hspace{1cm} (1.6)

• Output per year determines employment $L$:

$$\frac{Y_r(t)}{a(t)} = L(t)$$  \hspace{1cm} (1.7)

• The rate of employment $L/N = \dot{\lambda}$ determines the rate of change of the money wage—thus linking the physical sector to the monetary sector; in keeping with Phillips’s original intentions (and in contrast to most macroeconomic models), the wage change function includes a reaction to the rate of change of employment and the level of inflation, as well as a nonlinear reaction to the level of employment:

$$W(t) \cdot \left( P_h(\dot{\lambda}(t)) \right) + \omega \cdot \frac{1}{\dot{\lambda}(t)} \cdot \frac{d}{dt} \dot{\lambda}(t) + \frac{1}{P(t)} \cdot \frac{d}{dt} P(t) \Rightarrow \frac{d}{dt} W(t)$$  \hspace{1cm} (1.8)

• The money wage determines the rate of change of the price level $P$:

$$-\frac{1}{\tau_p} \left( P(t) - \frac{1}{1-\sigma} \cdot \frac{W(t)}{a(t)} \right) \Rightarrow \frac{d}{dt} P(t)$$  \hspace{1cm} (1.9)

• The monetary value of output $P \cdot Y_r$ minus wages $W \cdot L$ determines profit:

$$P(t) \cdot Y_r(t) - W(t) \cdot L(t) - (r_L \cdot F_L(t) - r_dF_d(t)) = \Pi(t)$$  \hspace{1cm} (1.10)

• The rate of profit $\frac{\Pi(t)}{P(t) \cdot K_r(t)} = \pi_r$ determines investment (and hence the amount of new credit money needed should desired investment exceed profit) and investment minus depreciation $\delta$ determines the rate of economic growth $g$:

$$\frac{\text{Inv}(\pi_r(t))}{v} - \delta = g(t)$$  \hspace{1cm} (1.11)

• The integral of investment determines the capital stock:

$$g(t) \cdot K_r(t) \Rightarrow \frac{d}{dt} K_r(t)$$  \hspace{1cm} (1.12)

• The rate of change of the employment rate is the rate of growth minus the rates of growth of labor productivity and population:

$$\dot{\lambda}(t) \cdot \left( g(t) - (\alpha + \beta) \right) \Rightarrow \frac{d}{dt} \dot{\lambda}(t)$$  \hspace{1cm} (1.13)

• Equations for growth in labor productivity and population complete the model:
\[ \alpha \cdot a(t) \Rightarrow \frac{d}{dt} a(t) \]
\[ \beta \cdot N(t) \Rightarrow \frac{d}{dt} N(t) \]

(1.14)

The rates of lending (A), debt-financed investment (D) and loan repayment (J) are modeled as nonlinear functions of the rate of profit, while the Phillips Curve is also a nonlinear function of the level of employment. The basic function used in all cases is a generalized exponential function where the arguments to the function are an \((x_c, y_c)\) coordinate pair, the function’s slope at that point \(s\), and its minimum \(m\):

\[
genexp(x, x_c, y_c, s, m) = (y_c - m) \cdot e^{\frac{(s - x)}{m}} + m
\]

(1.15)

The complete model is described by a set of ten differential equations:

\[
\frac{d}{dt} B_e(t) = \frac{F_e(t)}{\tau_e(\pi_e(t))} - B_e(t)
\]
\[
\frac{d}{dt} F_L(t) = \frac{B_e(t)}{\tau_e(\pi_e(t))} - \frac{F_L(t)}{\tau_L(\pi_L(t))} + \text{Inv}(\pi_e(t)) \cdot P(t) \cdot Y_R(t)
\]
\[
\frac{d}{dt} F_D(t) = \frac{B_e(t)}{\tau_e(\pi_e(t))} - \frac{F_d(t)}{\tau_L(\pi_L(t))} - r_d(t) \cdot F_L(t) + \text{Inv}(\pi_L(t)) \cdot P(t) \cdot Y_R(t) - W(t) \cdot \frac{Y_R(t)}{a(t)} + r_d(t) \cdot F_d(t) + \frac{W_R(t)}{\tau_w} \cdot \frac{B_e(t)}{\tau_s}
\]
\[
\frac{d}{dt} W_D(t) = W(t) \cdot \frac{Y_R(t)}{a(t)} + r_d(t) \cdot W_D(t) - \frac{W_D(t)}{\tau_w}
\]
\[
\frac{d}{dt} B_e(t) = r_d(t) \cdot F_L(t) - r_d(t) \cdot F_D(t) - r_d(t) \cdot W_D(t) - \frac{B_e(t)}{\tau_s}
\]
\[
\frac{d}{dt} K_R(t) = g(t) \cdot K_R(t)
\]
\[
\frac{d}{dt} \lambda(t) = \lambda(t) \cdot (g(t) - (\alpha + \beta))
\]
\[
\frac{d}{dt} W(t) = W(t) \cdot P_e(\lambda(t)) + \omega \cdot \frac{1}{\lambda(t)} \cdot \frac{d}{dt} \lambda(t) + \frac{1}{P(t)} \cdot \frac{d}{dt} P(t)
\]
\[
\frac{d}{dt} a(t) = \alpha \cdot a(t)
\]
\[
\frac{d}{dt} N(t) = \beta \cdot N(t)
\]

(1.16)

Given suitable initial conditions and parameter values, this highly nonlinear monetary model can generate the stylized facts of the last 20 years of macroeconomic data: an apparent “Great Moderation” in employment and inflation—which was actually driven by an exponential growth in private debt—followed by a “Great Recession” in which unemployment explodes, inflation turns to deflation, and the debt level—absent of bankruptcy and government intervention—goes purely exponential as unpaid interest is compounded.
As a complex systems model, the behavior of this system depends upon its initial conditions as well as upon its inherent dynamics. In Keen 2011 I used a set of initial conditions that resulted in both a Great Moderation and a Great Recession— with no change to the underlying parameters of the system—to indicate that this model fits Minsky’s criteria for a successful model of capitalism:

Can “It”—a Great Depression—happen again? And if “It” can happen, why didn’t “It” occur in the years since World War II? These are questions that naturally follow from both the historical record and the comparative success of the past thirty-five years. To answer these questions it is necessary to have an economic theory which makes great depressions one of the possible states in which our type of capitalist economy can find itself. (Minsky 1982, p. 5; emphasis added)
This model captures the macroeconomic experience of the last 2 decades far more effectively than any neoclassical model. However, the Holy Grail of economics has always been to model the complex dynamic process by which commodities are produced using other commodities and labor. In the next section I show that a structured extension of this corn economy model—with financial flows determining demand, and production modeled using Goodwin’s growth cycle—can generate a coherent dynamic monetary multisectoral model of production.

A dynamic monetary multisectoral model of production

First a strong caveat: this model is very tentative, and many refinements need to be made. However even in its tentative state, it shows that a monetary, dynamic multisectoral model of production can be constructed.

The model reproduces the structure of the preceding corn economy model, extended to multiple commodities in both production (with each sector needing to purchase inputs from other sectors proportional to its desired output level), and consumption. I also address one of the weaknesses of input-output analysis—that purchases within a sector are not explicitly shown—by the simple expedient of splitting each sector in two. There are 4 sectors in this simple “proof of concept” model (notionally Capital Goods, Consumer Goods, Agriculture and Energy).

The Godley Table for this system has 19 system states— Bank Reserve, Bank Equity and Worker Deposit accounts as in the single sectoral model, plus two Deposit and two Loan accounts per sector—and 16 financial operations—debt compounding, debt repaying, money relending and wages payment as in the single sectoral model, plus one intersectoral purchase for

---

**Figure 2: Simulation Results with uncalibrated constant parameter values**

Inflation, Unemployment and the Debt Ratio

- Red: Unemployment
- Blue: Inflation
- Black: Debt to Output

Percent

0 20 40 60

0 10 20

-20 -10 0 1 2

0 60

---
production and one for consumption per sector. A stylized representation of these flows is given in Table 3 (the intersectoral flows are only partially indicated).

Table 3: Stylized representation of multisectoral Godley Table

<table>
<thead>
<tr>
<th>Accounts</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Assets</td>
<td>Liabilities</td>
<td>Equity</td>
</tr>
<tr>
<td>Account</td>
<td>Bank Reserve</td>
<td>Sector 1 Loan</td>
<td>Sector 1 Deposit</td>
</tr>
<tr>
<td>Symbol</td>
<td>BR(t)</td>
<td>F_{L1}(t)</td>
<td>F_{D1}(t)</td>
</tr>
<tr>
<td>Compound Debt</td>
<td>A_1</td>
<td>A_2</td>
<td>B_1</td>
</tr>
<tr>
<td>Deposit Interest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>C_1</td>
<td>C_2</td>
<td>C_1 + C_2</td>
</tr>
<tr>
<td>Worker Interest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment K</td>
<td>E</td>
<td>-E</td>
<td></td>
</tr>
<tr>
<td>Intersectoral C</td>
<td></td>
<td>-F</td>
<td>F</td>
</tr>
<tr>
<td>Intersectoral A</td>
<td></td>
<td>-G</td>
<td>G</td>
</tr>
<tr>
<td>Intersectoral E</td>
<td></td>
<td>-H</td>
<td>H</td>
</tr>
<tr>
<td>Consumption K</td>
<td>I</td>
<td>-I</td>
<td></td>
</tr>
<tr>
<td>Consumption C</td>
<td>J</td>
<td>-J</td>
<td>J</td>
</tr>
<tr>
<td>Consumption A</td>
<td>K</td>
<td>-K</td>
<td>K</td>
</tr>
<tr>
<td>Consumption E</td>
<td>L</td>
<td>-L</td>
<td>L</td>
</tr>
<tr>
<td>Pay Interest</td>
<td>M</td>
<td>-M</td>
<td>M</td>
</tr>
<tr>
<td>Repay Loans</td>
<td>N</td>
<td>-N</td>
<td></td>
</tr>
<tr>
<td>Recycle Reserves</td>
<td></td>
<td>-O</td>
<td>O</td>
</tr>
<tr>
<td>New Money</td>
<td>P</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An extract from the actual Godley Table for this system (as implemented in Mathcad) is shown in Figure 3.
The rate of profit is now net of intersectoral purchases for each sector, and of course there is a different rate of profit in each sector. Intersectoral purchases of inputs differ for each sector, and are proportional to the labor input needed to produce the required output in each sector—signified by $\sigma_{i,j}$ where the first subscript represents the sector purchasing the inputs and the second the sector from which the inputs are purchased. Equation (1.17) shows the rate of profit formulae for the capital goods and consumer goods sectors:

$$\Pi_K(t) = P_K(t) \cdot Q_K(t) - W(t) \cdot L_K(t) - \sum_{S \in K} (\sigma_{KS} \cdot W(t) \cdot L_S(t)) - (r_L \cdot K_L(t) - r_D K_D(t))$$

$$P_K(t) \cdot K_K(t)$$

$$\Pi_C(t) = P_C(t) \cdot Q_C(t) - W(t) \cdot L_C(t) - \sum_{C \in C} (\sigma_{CS} \cdot W(t) \cdot L_S(t)) - (r_L \cdot C_L(t) - r_D C_D(t))$$

$$P_K(t) \cdot K_C(t)$$

As with the single sectoral model, behavior in five crucial areas is modeled as a nonlinear response to a relevant variable:  

- The rate of change of money wages as a function of the rate of employment;  
- The time constant in investment decisions $\tau_{PR}$ as a function of the rate of profit;  
- The time constant in loan repayment as a function of the rate of profit;  
- The time constant in money relending as a function of the rate of profit;  
- The time constant in new money creation as a function of the rate of profit;  

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7 I have had this described to me as “an assumption of irrational behavior” by neoclassical economists who are accustomed to the assumption of rational expectations. I find this accusation bizarre, since nothing could be more irrational than to assume that agents in a complex system can accurately predict its future course—and yet this is precisely what “rational expectations” entails. In my models, people react rationally to the information they believe is relevant and that they have at hand, but they cannot and do not predict the future course of the economy—or if they try to, their predictions will be wrong. Assuming fallibility is not the same as assuming irrationality!
Table 4: Parameters for Behavioral Functions

Wage Change Function

<table>
<thead>
<tr>
<th>Rate of change of Money Wages percent</th>
<th>genexp(x, 92%, 0%, 1, -4%)</th>
</tr>
</thead>
</table>

Employment Rate (percent normal participation rate)

Investment as function of rate of profit

<table>
<thead>
<tr>
<th>Rate of profit</th>
<th>genexp(x, 3%, 7 years, -200, 5 years)</th>
</tr>
</thead>
</table>

Time constant for doubling of capital stock

Loan repayment as function of rate of profit

<table>
<thead>
<tr>
<th>Rate of profit</th>
<th>genexp(x, 3%, 10 years, 100, 3 years)</th>
</tr>
</thead>
</table>
With the purchases of intermediate inputs taken care of in the monetary demand component of the model, production in each sector is modeled as lagged response to installed capital, and employment is a lagged response to output. The functions for the consumer goods sector, which are representative of those for the other sectors, are shown in Equation (1.18):

\[ \text{genexp}(x, 3\%, 7 \text{ years}, -300, 1 \text{ year}) \]

\[ \text{genexp}(x, 3\%, 10 \text{ years}, -300, 2 \text{ years}) \]
\[
\frac{d}{dt} K_C(t) = \frac{F_{DK}(t)}{\tau_{pr}(\pi_{c}(t)) \cdot P_{K}(t)} - \gamma K_C(t) \quad \text{Capital Stock}
\]
\[
\frac{d}{dt} Q_C(t) = -\frac{1}{\tau_{OC}} \left( Q_C(t) - \frac{1}{v_C} \cdot K_C(t) \right) \quad \text{Output}
\]
\[
\frac{d}{dt} L_C(t) = -\frac{1}{\tau_{LC}} \left( L_C(t) - \frac{1}{a_C(t)} \cdot Q_C(t) \right) \quad \text{Labor}
\]
\[
\frac{d}{dt} P_C(t) = -\frac{1}{\tau_{PC}} \left( P_C(t) - \frac{W(t)}{a_C(t) \cdot (1 - s_C)} \right) \quad \text{Price Level}
\]
\[
\frac{d}{dt} a_C(t) = \alpha \cdot a_C(t) \quad \text{Labor Productivity}
\]

The full model is a system of \((2 \cdot 2 \cdot n + 3) + 5 \cdot n + 1\) differential equations, where \(n\) is the number of sectors, and the first set of terms specifies the equations in the financial sector, the second the equations in production, and the final equation is for population growth. In this sample 4-sector model, this results in a system of 40 nonlinear ODEs.

**Results**

The rate of profit varied between sectors, and, once the system had settled into its limit cycle, ranged from 0.4% p.a. and 8.7%.
The aggregate real rate of economic growth varied between minus 1 and plus 5 percent p.a., and growth followed a sawtooth pattern:
This shape corresponds with the stylized nature of the business cycle, as Blatt observed:

In the real world, upswings are slow; downswings go with an almighty rush. In the words of Galbraith:

“The usual image of the business cycle was of a wavelike movement, and the waves of the sea were the accepted metaphor… The reality in the nineteenth and early twentieth centuries was, in fact, much closer to the teeth of a ripsaw which go up on a gradual plane on one side and drop precipitately on the other…” (Blatt 1983, pp. 203-204, citing Galbraith 1975, p. 104)

The growth rate and the debt to output level moved together, and the debt ratio cycled between 50 and 110 percent of GDP.
The distribution of income was realistic, though the dynamics were rather more volatile than in actual data:
The rate of inflation was unrealistic, with a minimum of 8 percent p.a. and a maximum of 45 percent.
These last two empirical weaknesses probably reflect the specification for the Phillips curve, and the tendency of the model to operate in over-full employment (defined as a ratio of 1 in this simple model) given the parameters used for capitalist and banker behavior.

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8 In this simple model, the population level effectively meant the available workforce—rather than the total population, with a large proportion of that being not of working age. I also used a simple single factor Phillips Curve, rather than the more realistic 3-factor function used in the preceding single sector model.
Finally, financial dynamics were an essential part of this model: money is far from neutral in this model (and in the real world). Periods of falling economic growth coincided with an increase in bank reserves, and a decline in the level of loans.
Conclusion

Though this preliminary model has many shortcomings, the fact that it works at all shows that it is possible to model the dynamic process by which prices and outputs are set in a multisectoral economy. The failure of the neoclassical school to achieve this objective—which it has had since the time of Walras—may relate to the abstractions it made with the intention of making this process easier to model. These devices—everything from Walras’s tatonnement, to ignoring the role of money—may in fact be why they failed. The real world is complex and the real economy is monetary, and complex monetary models are needed to do it justice.

Given the complexity of this model and the sensitivity of complex systems to initial conditions, it is rather remarkable that an obvious limit cycle developed out of an arbitrary set of parameter values and initial conditions—with most (but by no means all) variables in the system keeping within realistic bounds. A conjecture is that this limit cycle is a manifestation of the well-known instability of an input-output matrix (Jorgenson 1960; Jorgenson 1960; Jorgenson 1961; Jorgenson 1961; Hahn 1963; Blatt 1983; Fleissner 1990; Heesterman 1990; Johnson 1993), combined with nonlinear relations that reverse the instability properties of the system as it diverges from its equilibrium. This conjecture was first made by Blatt in a discussion of both the historical evidence of the business cycle and the dual instability of the equilibrium growth path:

At this stage of the argument, we feel free to offer a conjecture: The repeated development of an unstable state of the economy is associated with, and indeed is
an unavoidable consequence of, the local instability of the state of balanced growth. (Blatt 1983, p. 161)

The presence of monetary buffers—in the guise of deposit accounts—surely also plays a role in the system’s capacity, despite its instability, to stay within realistic bounds, in contrast to most (if not all) other dynamic multisectoral models.

I doubt that Kuznets would have been surprised by the failure of equilibrium-oriented attempts to build dynamic multisectoral models of economic growth, since he argued long ago that dynamics had to be different to statics, and in particular that the fetish with equilibrium had to be abandoned:

According to the economists of the past and to most of their modern followers, static economics is a direct stepping stone to the dynamic system, and may be converted into the latter by the introduction of the general element of change… According to other economists, the body of economic theory must be cardinally rebuilt, if dynamic problems are to be discussed efficiently…

… as long as static economics will remain a strictly unified system based upon the concept of equilibrium, and continue to reduce the social phenomenon to units of rigidly defined individual behavior, its analytic part will remain of little use to any system of dynamic economics… the static scheme in its entirety, in the essence of its approach, is neither a basis, nor a stepping stone towards a proper discussion of dynamic problems. Kuznets, S. (1930, pp. 422-428, 435-436; emphasis added)

Yet the static approach—masquerading as dynamics via word games such as using the moniker “Dynamic Stochastic General Equilibrium” to describe bastardized Ramsay-Solow equilibrium growth models—still dominate economics, even after the continuing disaster of the crisis of 2007. Part of the reason for this persistence, I believe, is the seductive simplicity of the “Marshallian Cross” that forms the basis of education in economics: it conforms to Henry Menchen’s aphorism that “Explanations exist; they have existed for all time; there is always a well-known solution to every human problem—neat, plausible, and wrong”. For economics to escape the trap of static equilibrium thinking, we need an alternative foundation methodology that is neat, plausible, and—at least to a first approximation—right.

I offer this model and the tools used to construct it as a first step towards such a neat, plausible and generally correct approach to macroeconomics. A colleague has implemented the Godley Table method for building a dynamic model of financial flows in a prototype dynamic modeling program QED, which is freely downloadable from my blog. A Mathematica implementation is being developed as part of a project with the CSIRO and it will also be freely available from my blog when it is completed. The ultimate objective is to develop a standalone dynamic monetary macroeconomic modeling tool that is more suited to financial flows than existing systems dynamics programs like Simulink, Vensim and Vissim.

10 Go to http://www.debtdeflation.com/blogs/qed/; QED stands for “Quesnay Economic Dynamics”.
11 The Commonwealth Scientific and Industrial Research Organisation is Australia’s public research institution.
The global economy was blindly led into our current financial crisis by an economics profession that had deluded itself into the belief that such phenomena cannot occur. Hopefully, during this crisis, monetary macroeconomic dynamics will finally supplant the static method against which Kuznets inveighed so eloquently at the start of capitalism’s previous great financial crisis.

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16.


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