Keynes's 'revolving fund of finance' and transactions in the Circuit

By Steve Keen[1]

Keynes's primary motivation in writing "Alternative theories of the rate of Interest" and "The "ex-ante" theory of the rate of interest" was to counter attempts by Ohlin and others to recast his liquidity preference theory as no more than a supply and demand model of the determination of the rate of interest. This rearguard action was ultimately unsuccessful, given the profession's ultimate acceptance of Hicks's IS-LM analysis as a summary of the *General Theory*. However, it also had a positive outcome, as tussling with Ohlin's arguments led Keynes to propose that investment finance was "an additional demand for money" (Keynes 1937b: 247) to the *General Theory*'s triumvirate of transactions, precautionary and speculative demands.

Keynes's musings on the interplay between firms who wish to borrow to finance investment, and banks that provide that finance, is prescient of, and of course partly inspired, the Circuitist School's later contribution. But Keynes's less formal logic also reached some conclusions contrary to current Circuitist belief. Keynes was correct on these points, while recent Circuitist literature is in error. Notwithstanding this however, the contributions of Graziani *et alia* on the nature of a monetary economy are essential to the development of a proper model of Keynes's "revolving fund of liquid finance" (Keynes 1937c: 666).

THE REVOLVING FUND

Keynes identifies three sources of confusion between himself and Ohlin, Hicks and Robertson (Keynes 1937b: 241-246); the third of these—a confusion between the money needed to initiate an investment, and the money needed while investment is actually proceeding—led to the development of the concept of a finance demand for money:

> I proceed to the third possible source of confusion, due to the fact (which may deserve more emphasis than I have given it previously) that an investment *decision* (Prof. Ohlin's investment *ex-ante*) may sometimes involve a temporary demand for money before it is carried out, quite distinct from the demand for active balances which will arise as a result of the investment activity whilst it is going on. (Keynes 1937b: 246)

Keynes emphasizes that, if a planned investment is to be turned into an actual one, then the investor will have a need for money that precedes the investment itself:

> Planned investment—i.e. investment *ex-ante*—may have to secure its "financial provision" *before* the investment takes place; that is to say, before the corresponding saving has taken place… There has, therefore, to be a technique to bridge this gap between the time when the decision

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> to invest is taken and the time when the correlative investment and saving actually occur. (Keynes 1937b: 246)

This finance could be secured either by new equity or new bank debt. In either case, there will be an imbalance between the market's commitments to finance these ventures, and actual savings at that point in time, which generates a "finance demand for money". Keynes argues that this should be considered a fourth, additional motive for desiring money in addition to the transactions, precautionary and speculative motives detailed in the *General Theory*:

Investment finance in this sense is, of course, only a special case of the finance required by any productive process; but since it is subject to special fluctuations of its own, I should (I now think) have done well to have emphasised it when I analysed the various sources of the demand for money. (Keynes 1937b: 247)

Keynes's discussion of how this demand might be met strengthens Dow's case, that Keynes viewed the money supply as endogenous (Dow 1995). Though he observes that additional finance demand for money might drive up the rate of interest—which is consonant with a fixed, exogenously determined money stock—he also countenances that the banking system might meet this demand with an additional supply—which implies an endogenous process of money creation:

Now, a pressure to secure more finance than usual may easily affect the rate of interest through its influence on the demand for money; and *unless the banking system is prepared to augment the supply of money*, lack of finance may prove an important obstacle to more than a certain amount of investment decisions being on the tapis at the same time. (Keynes 1937b: 247; emphasis added)

Keynes continues that the decision to supply money as finance for investment is an important determinant of the level of economic activity. Thus while he rejected the "classical" view that savings determined investment, he argued that *finance determines investment, and investment in turn determines savings*.

> It is the supply of available finance which, in practice, holds up from time to time the onrush of 'new issues.' But if the banking system chooses to make the finance available and the investment projected by the new issues actually takes place, the appropriate level of incomes will be generated out of which there will necessarily remain over an amount of saving exactly sufficient to take care of the new investment. (Keynes 1937: 248)

In making this case, Keynes also states unambiguously that banks control the supply of money:

> The control of finance is, indeed, a potent, though sometimes dangerous, method for regulating the rate of investment (though much more potent when used as a curb than as a stimulus). Yet this is only another way of expressing the power of the banks through their control over the supply of money—i.e. of liquidity. (Keynes 1937: 248)

Money is thus an endogenous variable, with its determination involving both the desire by firms to invest, and the willingness of banks to lend. Keynes starts his consideration of this process with a constant level of investment—i.e., with a steady stream of investment projects coming forward over time, so that the rate of change of aggregate investment with respect to time is zero. In this case, Keynes argues that a constant stream of investment can be financed by a fixed pool of money, which turns over continuously:

> If investment is proceeding at a steady rate, the finance (or the commitments to finance) required can be supplied from a revolving fund of a more or less constant amount, one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment. (Keynes 1937b: 247)

This implies that a constant level of economic activity can be sustained by a constant stock of money—since investment in turn determines the level of income, and a constant level of gross investment implies a constant capital stock. Rising investment, on the other hand, implies rising capital and rising output, and here Keynes argues that there will be a rising demand for money for finance: "if decisions to invest are (e.g.) increasing, the extra finance involved will constitute an additional demand for money" (Keynes 1937b: 247).

As noted above, Keynes countenances that this demand could put upwards pressure on the rate of interest, if banks did not generate more money. But it could also lead to banks increasing the money supply "if the banking system chooses to make the finance available". In tranquil times, banks would willingly supply additional finance when firms desired a rising level of investment, and this in turn would cause rising incomes over time. The demand for money would thus call forth its supply.

Keynes concludes with observations about the tendency of economists to confuse finance and saving, and stocks and flows. "'Finance'", he emphatically declared,

> has nothing to do with saving. At the 'financial' stage of the proceedings no net saving has taken place on anyone's part, just as there has been no net investment. 'Finance' and 'commitments to finance' are mere credit and debit book entries, which allow entrepreneurs to go ahead with assurance. (Keynes 1937b: 247).

Keynes's conjecture that confusion between stocks and flows was the source of important errors in monetary theory is worth quoting at length:

> It is possible, then, that confusion has arisen between credit in the sense of 'finance,' credit in the sense of 'bank loans' and credit in the sense of 'saving.' I have not attempted to deal here with the second. *It should be observed that a confusion between the first and the last would be one between a flow and a stock*. Credit, in the sense of 'finance,' looks after a flow of investment. It is a revolving fund which can be used over and over again. It does not absorb or exhaust any resources. The same 'finance' can tackle one investment after another. But credit, in Prof. Ohlin's sense of 'saving,' relates to a stock. Each new net investment has new net saving attached to it. The saving can be used once only. It

relates to the net addition to the stock of actual assets. (Keynes 1937b: 247; emphasis added)

Keynes's concept of a finance demand for money thus provides a link between a flow of demand for credit money, and the stock of credit money that is needed to meet that flow demand, given the time lags in the economy.

Unlike Keynes, the Circuitist School has attempted to deal with "credit in the sense of 'bank loans'". In so doing, they have reached several conclusions that implicitly or explicitly contradict Keynes.

Keynes implicitly argues that capitalists could make aggregate money profits, after borrowing money at positive rates of interest, when he speaks of "one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment". In contrast, Circuitists explicitly allege that capitalists cannot make aggregate monetary profits, even if the rate of interest is zero:

"in the basic circuit approach (describing a closed economy with no government expenditure), firms in the aggregate can only obtain the wage bill they advanced to workers (w_N) and, as a result, it is impossible for all firms to obtain money profits." (Bellofiore et al. 2000: 410)[1]

Keynes argues that constant economic activity could be supported with a constant stock of money, regardless of how workers allocated their wages. Circuitists claim that a constant level of activity requires an increasing stock of money if workers save, since with part of the borrowed money saved, firms are unable to repay their bank loans in full:

> If, as is likely to be the case, firms wish to continue their activities, they have to renegotiate bank loans equal to the net stock of money in addition to any lending necessary to start a new production process. (Fontana 2000: 35)

Crucially, Keynes sees money turning over indefinitely in "revolving fund of liquid finance"—so that money, once created, exists forever (though he did not consider the issue of bankruptcy). On the other hand, in Circuitist literature, money is "destroyed" when loans are repaid:

> "To the extend that bank debts are repaid, an equal amount of money is destroyed" (Graziani 2003: 29-30)

In all these points of contradiction, Keynes is correct and the Circuitists are wrong, for the reason Keynes gave in 1937: Circuitists, like so many economists before them, have confused stocks with flows. However, Circuitist insights into the nature of money, and of exchange in a monetary economy, play a crucial role in turning Keynes's accurate verbal insights into a workable mathematical model of a monetary production economy.

THE CANONICAL CIRCUITIST INSIGHTS

The three key contributions of the Circuitist School are:

- The proposition that a true monetary economy cannot use a commodity as money;

- The insight that exchanges in a monetary production economy are three-sided, single commodity transactions; and
- A logical definition of money that is free of the customary confusions that arise from defining money in terms of different types of bank deposits.

The first proposition is derived from the simple observation that "an economy using as money a commodity coming out of a regular process of production, cannot be distinguished from a barter economy" (Graziani 1989: 3). From this it follows that true money is a token, which in turn gives rise to two further conditions, that:

> the use of money must give rise to an immediate and final payment and not to a simple commitment to make a payment in the future; and
>
> the use of money must be so regulated as to give no privilege of seigniorage to any agent. (Graziani 2003: 60)

These conditions lead to the second fundamental insight, that all sales in a monetary economy involve three parties: a seller, a buyer, and a bank which transfers the requisite number of units of account from the buyer's account to the seller's.

These in turn provide a definition of money that enables it to be clearly distringuished from credit—another confusion that Keynes notes. Money is as a unit of account whose transfer is accepted as final payment in all commodity and service exchanges; credit, on the other hand, enables a commodity or service exchange to occur, but involves a continuing debtor-creditor relationship between the buyer and the seller.

CIRCUITISTS AND CHARTALISTS

The State plays no necessary role in the above definition of money—though Circuitists of course acknowledge the existence of "fiat" money, and generally accept the Chartalist or state theory of money position with respect to the origins of money and its modern legal framework (see for example Graziani 2003: 78-80). However, this School has attempted to build models which at the outset have no government sector—nor any explicit role for the Central Bank (Graziani 2003: 26-32). In this sense, the Circuit approach conflicts with the Chartalist argument that "It is thus impossible to separate the theory of money from the theory of the state" (Wray 2000: 50).

From the Circuitist point of view, the production and enforcement of a unit of account by a tax-levying state is an embellishment to its fundamental concept of money. The Circuitist starting point of a pure credit economy is thus arguably closer to the essential nature of money, even if so-called "State Money" is the universal norm today, and even State enforcement of monetary obligations may be the only viable way to sustainably meet Graziani's anti-seignorage condition in the real world.

However, the failure to date of Circuitists to produce a coherent model of endogenous money could have implied that the Chartalist position was correct, in that a tax-levying state was indeed an essential component of a functional model of money. In fact, as I show below, a functional model of a monetary production economy can be built without either a government sector or a central bank, so long as transfers between private bank accounts are accepted as making final settlement of debts between buyers and sellers.

THE BASIC CIRCUITIST MODEL

Graziani 2003 presents a canonical version of the Circuitist verbal model of a monetary production economy. The model is described as having four classes of agents—"the central bank, commercial banks, firms and wage earners" (26-27) —but despite this, the central bank is given no role in the model itself. The actual model therefore has only three agents.[2]

The model's monetary dynamics commence with "A decision ... by the banks to grant credit to firms, thus enabling them to start a process of production" (27). Graziani argues that the amount of credit demanded by the firms (and supplied by the banks) equals the wage bill for the planned level of production.

Using the borrowed money, capitalists pay workers and put them to work to produce commodities. These are then sold, with consumer goods being sold to workers and investment goods to other capitalists (sales to bankers appear later).

Spending by workers on consumer goods (and also purchases of corporate bonds by workers) return money to the firms, who can then use this money to repay their debt to banks. This repayment of debt destroys money: "To the extend that bank debts are repaid, an equal amount of money is destroyed" (29-30).

The repayment of debt closes the circuit, but this only happens "If wage earners spend their incomes entirely" (including on purchases of corporate bonds). However if they don't, then dilemmas arise:

> If instead wage earners decide to keep a portion of their savings in the firm of liquid balances, firms are unable to repay their bank debt by the same amount. (30)

The next cycle, if it involves an identical scale of production, therefore requires new money, so that the money supply must increase to finance a constant scale of production. The new quantity of money in this second circuit "will be equal to the wage bill plus the new liquid balances set aside by wage earners at the end of the previous cycle" (31).

The above, however, omits the problem of interest on debt! Graziani acknowledges this—in contrast to some Circuitist papers that abstract from the problem, in a manner that is embarrassingly reminiscent of the neoclassical approach to logical conumdrums (Bellofiore et al. 2000: 410—footnotes 8 and 9). It appears that firms are unable to pay interest:

> even in the most favourable case [corresponding to workers spending all their wages], the firms can only repay in money the principal of their debt and are anyhow unable to pay interest. (31)

The solution he proffers, in a monetary model, is a "real" one, that banks are paid in commodities rather than money: "the only thing they can do is to sell part of their product to the banks, which is tantamount to saying that interest can only be paid in kind" (31).

At least bankers get their hands on the physical loot: capitalists, it seems, end up with neither goods nor money. Money profits in the aggregate are zero, and "profits earned by one firm may simply be the mirror image of inefficiencies and consequent losses incurred by other firms" (32).

A DYNAMIC MODEL OF THE CIRCUIT

Starting from precisely the same foundation, I reach contrary conclusions on almost every point above, and conclude instead that Keynes's 1937 insights were correct. A constant level of production can be financed with a constant stock of money (see also Andresen 2006); firms can easily pay the interest on debt with money; and firms in the aggregate earn money profits. Money is not destroyed by the repayment of debt (though bank deposits are "destroyed" by loan repayment, and the stock of money available for transactions at any one time is reduced); workers can have positive bank balances without forcing firms to make losses; and, though it is related to the wage bill, the initial amount borrowed is in fact far smaller.

These contrary conclusions arise simply from applying the correct form of mathematical analysis to the Circuitist school's logical insights into the nature of a monetary production economy. The Circuit is fundamentally dynamic, and can therefore only be properly understood using dynamic analysis. Mathematical dynamics are essential here, partly because the interrelations between entities in a dynamic model are easily mis-specified in verbal analysis, and especially because it is easy, in a verbal exposition, to confuse stocks and flows. In what follows, I construct a skeletal dynamic mathematical model of the Circuit, using balance sheets in which all entries are flows.

The model is, I stress, *deliberately* skeletal: causal factors of financial flows that are clearly variables in the real world are treated as constants—with the intention that these will indeed be made variables in a later model. However, just as much is learnt in anatomy by studying skeletons, much can be learnt about the actual monetary systems by studying a stylized system in which the causes of financial instability are absent.

Graziani's model has three classes of agents—firms, bankers, and workers. Since this is a monetary economy, all three classes have deposit accounts which I indicate as F_D, B_D and W_D respectively. Prior to the making of a loan, all three accounts have zero balances, and firms' debt to banks F_L is likewise zero (this is **not** a bank account as such: it does not contain money, nor can money be paid into it, but it instead records the outstanding obligation of the firms to the banks; it is, therefore, a record of account). This "ab initio" situation is shown in Table 1.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Initial values	**0**	**0**	**0**	**0**

Table 1: Initial conditions prior to loan

In step one of the model, banks make loans to the firms. Since this is credit money, a debt obligation is created between the firms and banks along with the creation of money. Using L to signify the magnitude of the loan, this results in the situation shown in Table 2. This clearly embodies the direct and causal “loans create deposits” perspective of endogenous money.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Start of loan	**L**	**L**	**0**	**0**

Table 2: Loan issued

A loan generates an obligation to pay interest to the lender, while a deposit obligates the bank to pay interest to the depositor. I use r_L for the rate of interest on loans and r_D for the rate on deposits, (where $r_L > r_D$). These obligations are shown in Table 3.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Obligations initiated by loan	$+r_L F_L$	$+r_D F_D$	**0**	**0**

Table 3: Loan and deposit obligations

We now move from the loan obligations to the flows which must occur out of accounts in the system—since there is no other source of money. The firms must therefore pay the loan interest obligation out of their deposit account F_D, while the bank must pay its deposit interest obligation out of its deposit account B_D.

The flows occur between these two deposit accounts, and the payment of loan interest is recorded on the asset side of the ledger, so that the firms’ debt remains constant at the level of the initial loan L. Since the interest payments flow between the firm and banker deposit accounts, the overall sum of deposit accounts also stabilises at L; but since $r_L > r_D$, the balance shifts from the firms deposit account to the bankers over time. This dynamic is shown in Table 4.

Bank Assets & Liabilities					
Flows	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Sum**
Interest flows initiated by loan	$+r_L F_L$ $- r_L F_L = 0$	$+r_D F_D$ $- r_L F_L$	$+r_L F_L$ $- r_D F_D$	**0**	**0**

Table 4: Payment of interest

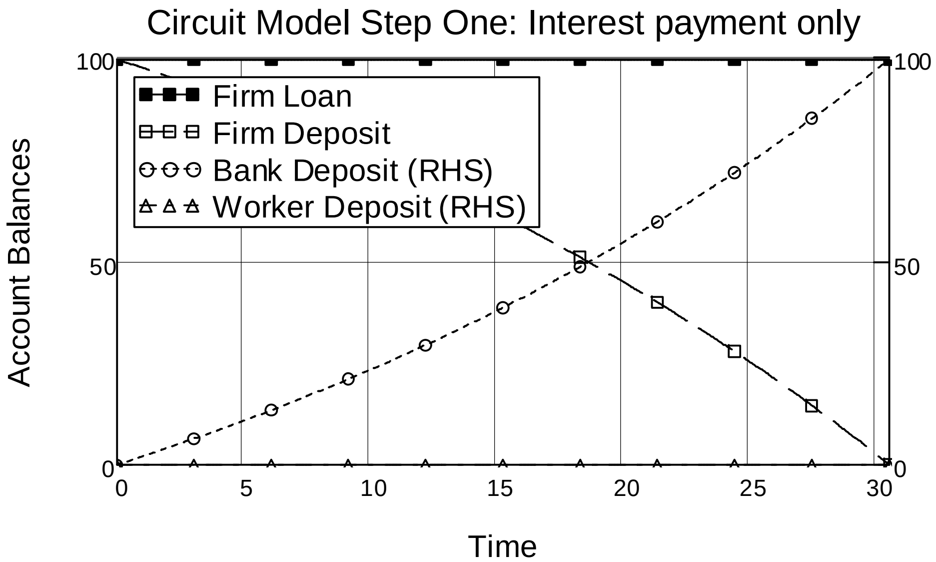
Equation (0.1) states this incomplete system as a set of coupled ODEs. It is obvious that the level of debt will remain constant (at the initial value L), as will the sum of deposit accounts, but the money in the firms' account will over time be transferred to the banks'. At some point, firms' deposit accounts will turn negative—which is of course an unsustainable situation.

$$\begin{aligned} \tfrac{d}{dt}F_L &= 0 \\ \tfrac{d}{dt}F_D &= r_D F_D - r_L F_L \\ \tfrac{d}{dt}B_D &= r_L F_L - r_D F_D \\ \tfrac{d}{dt}W_D &= 0 \end{aligned} \qquad (0.1)$$

Figure 1 shows a simulation of this system. Given the set of example parameter values (L=100, r_L=5%, r_D=3%) while the outstanding loan and the sum of deposit accounts remain at 100 throughout, all the money has been transferred from the firms' deposit account to the bankers' after 30.5 years.

Given	Initial values	Flow dynamics
Firm loan account	$F_L(0) = L$	$\frac{d}{dt}F_L(t) = r_L \cdot F_L(t) - r_L \cdot F_L(t)$
Firm deposit account	$F_D(0) = L$	$\frac{d}{dt}F_D(t) = r_D \cdot F_D(t) - r_L \cdot F_L(t)$
Bank deposit account	$B_D(0) = 0$	$\frac{d}{dt}B_D(t) = r_L \cdot F_L(t) - r_D \cdot F_D(t)$
Worker deposit account	$W_D(0) = 0$	$\frac{d}{dt}W_D(t) = 0$

$$\begin{pmatrix} F_L \\ F_D \\ B_D \\ W_D \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} F_L \\ F_D \\ B_D \\ W_D \end{pmatrix}, t, Y\right]$$



$F_D(Y) = 0$ $\quad B_D(Y) = 100$ $\quad W_D(Y) = 0$ $\quad F_D(Y) + B_D(Y) + W_D(Y) = 100$

Figure 1: Simulation of interest payment only model in Mathcad

This outcome possibly explains why Circuitists have been loathe to acknowledge the need to pay interest in their models of the monetary circuit: the situation seems hopeless for firms. However, this is only because firms have not yet done anything with the borrowed money. In fact, it has been borrowed to finance production, which involves both buying inputs from other firms, and paying wages to workers. This in turn is done in order to evoke a stream of purchases from other firms, workers and bankers from which the firms hope to make a net profit.

The issue of production, and the transactions enabling it and emanating from it, is another area of great confusion in Circuitist writings. The key confusion is one of stocks and flows, starting from the proposition that the size of the initial loan (the *stock* L) is equal to the wage payments needed to hire the workforce (a *flow*). Instead, the wage bill is related, not to the initial loan, but to the rate of outflow of money from firms' deposit accounts that is used to pay wages. Calling this rate of outflow w, an amount $w.F_D$ is transferred per unit of time (per year in this model) from firms to workers as wages.

Bank Assets & Liabilities					
Flows	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Sum**
Wage flow to initiate production		**$-w. F_D$**		**$+w. F_D$**	**0**

Table 5: Spending to finance production

The relationship between money and wages is thus not "the credit initially granted [L, a stock] is totally turned into wages [$w.F_D$, a flow]" (Graziani 2003: 29). Instead, in this skeletal model, wages equal a constant times the balance in the firms' deposit account.[3] Given the relationship between the initial loan and the balance in the firms' account, the annual wages paid can be substantially greater than the initial loan.

With workers now having positive bank balances, they too are receipients of interest income. Though in the real world workers normally get lower deposit rates than firms, for simplicity I will use the same rate of interest r_D here. A flow of $r_D.W_D$ is therefore deducted from the bankers' account and deposited into the workers' account.

Bank Assets & Liabilities					
Flows	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Sum**
Interest income flows from wages			$- r_D . W_D$	$+r_D . W_D$	**0**

Table 6: Incomes from production

To complete the model, we have to include the flow of transactions from workers and bankers to capitalists that purchase the goods flowing (implicitly in this model) in the opposite direction. Here I use ω for the rate at which spending flows from workers' deposit accounts to firms', and β for the corresponding rate of spending by banks. The amounts $\omega \cdot W_D$ and $\beta \cdot B_D$ are therefore deducted from workers and banks accounts respectively and credited to the firms' account.

The basic model is finally complete, and as shown by the sum column of the Social Accounting Matrix, all transactions are properly accounted for and sum to zero—so that money is neither created nor destroyed. The components of the basic coupled ODE model can now be read down the columns of the final 4 rows of Table 7.

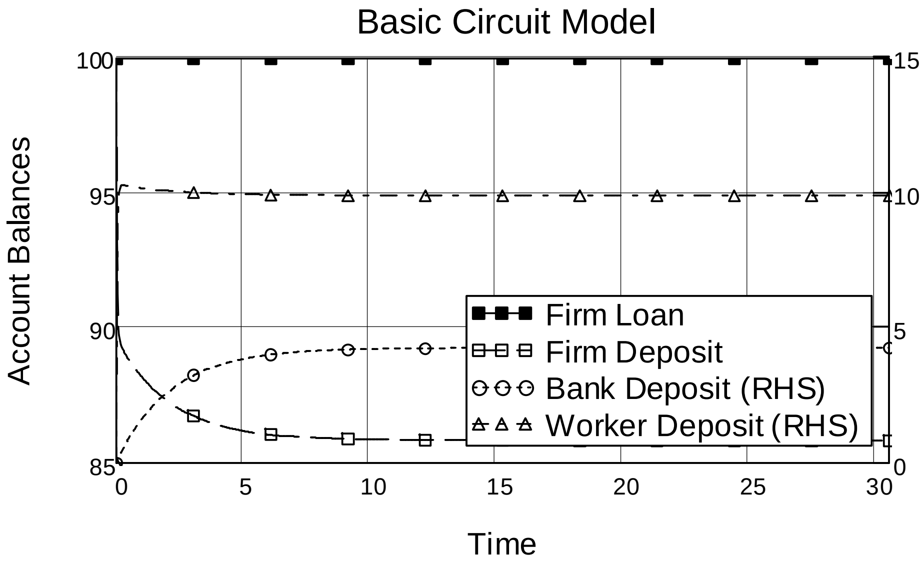
Bank Assets & Liabilities					
Flows	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Sum**
Interest flows initiated by loan	**0**	**$+r_D.F_D$ $-r_L.F_L$**	**$+r_L.F_L$ $-r_D.F_D$**	**0**	**0**
Wage flow to initiate production		**$-w.F_D$**		**$+w.F_D$**	**0**
Interest income flows from wages			**$-r_D.W_D$**	**$+r_D.W_D$**	**0**
Flows from sale		$+\omega\cdot W_D$ $+\beta\cdot B_D$	$-\beta\cdot B_D$	$-\omega\cdot W_D$	**0**

Table 7: Transactions complete the basic model

In coupled ODE form, the model is as shown in Equation (0.2).

$$\begin{aligned}
&\tfrac{d}{dt}F_L = 0\\
&\tfrac{d}{dt}F_D = \left(r_D F_D - r_L F_L\right) - w\cdot F_D + \left(\omega\cdot W_D + \beta\cdot B_D\right)\\
&\tfrac{d}{dt}B_D = \left(r_L F_L - r_D F_D\right) - r_D\cdot W_D - \beta\cdot B_D\\
&\tfrac{d}{dt}W_D = w\cdot F_D + r_D\cdot W_D - \omega\cdot W_D
\end{aligned} \quad (0.2)$$

The model can now be simulated (see Figure 2; the additional parameter values used here are w=3, $\omega = 26$ and $\beta = 0.5$), and since it is a linear model, its equilibrium can also be derived symbolically (see equation (0.3))



$F_D(Y) = 85.83$ $B_D(Y) = 4.255$ $W_D(Y) = 9.915$ $F_D(Y) + B_D(Y) + W_D(Y) = 100$

Figure 2: Basic Circuit model

As is now obvious, the basic Circuitist model with a single injection of endogenous money is consistent with sustained economic activity over time—contradicting the Circuitists since an increasing supply is not needed to sustain constant economic activity, and confirming Keynes 1937b (see also Andresen 2006). However, the amounts shown here are transaction account balances: we do not yet know whether these are compatible with sustained incomes over time.

$$\begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \end{bmatrix} = \begin{bmatrix} L \\ \dfrac{L\cdot(\omega - r_D)\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \\ \dfrac{L\cdot(r_L - r_D)}{\beta - r_D} \\ \dfrac{L\cdot w\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 100 \\ 85.83 \\ 4.255 \\ 9.915 \end{bmatrix} \qquad (0.3)$$

Income dynamics

Fortunately, two income flows are easily associated with particular transactions in equation (0.2): wages and interest income. Annual wages are equal to $w\cdot F_D$ and gross bank interest income is $r_L F_L$ (257.489 and 5 per annum respectively in this simulation). Wages and interest income are thus positive and sustained; what about profits?

To reveal profits, we need to consider what the term w represents. As well as being equivalent to wages, it also represents that part of the net surplus from production that accrues to workers. The net surplus—in monetary terms—itself depends on how rapidly money invested in production returns to firms. In Marx's terms, it represents the time lag between extending M and receiving M+ (assuming, as I do in this skeletal model, that the process occurs smoothly). This could be a period of, say, 4 months between financing production and receiving the complete proceeds of sale of output—again something that would be a variable in a more complex model. There are thus two components to w: the share of the net surplus (in Sraffa's sense of the surplus, in which wages and profits are entirely paid out of the net surplus from the input-output process) from production going to workers, and the rate of turnover from M to M+, given by technical conditions of production and the time taken for the sale of physical commodities. I use s for the share of surplus accruing to the owners of firms (so that the share going to workers is thus 1-s), and P for the lag between M and M+.[4] We therefore have the relation given by equation (0.4):

$$w = (1-s)\cdot P \quad (0.4)$$

With w set to 3 in the simulation above, a hypothetical value of s of 0.4 (which corresponds to a "rate of surplus value" in Marx's terms of 67%) yields a value for P of 5 (which means that the lag between spending M and making M+ is $1/5^{th}$ of a year or 2.4 months). The monetary value of net output per annum is thus $P.F_D$ (which equals 429.15 in equilibrium, given the parameter values in the model) which is split between workers and the owners of firms in the ratio (1-s):s. In this debt-finance only model, the owners of firms then have to pay interest on their outstanding debt to banks. Using Π, W and I to signify profits, wages and interest income respectively, the income flows of the model in equilibrium are:

$$\begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} s\cdot P\cdot \dfrac{L\cdot(\omega - r_D)\cdot(\beta - r_L)}{((1-s)\cdot P + \omega - r_D)\cdot(\beta - r_D)} \\ r_D\cdot L \\ (1-s)\cdot P\cdot \dfrac{L\cdot(\omega - r_D)\cdot(\beta - r_L)}{((1-s)\cdot P + \omega - r_D)\cdot(\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 166.66 \\ 5 \\ 257.49 \end{bmatrix} \quad (0.5)$$

Firms thus do make net profits, which, though related to the size of the initial loan, can be substantially larger than this amount (and profits are substantially larger than the servicing cost of debt). Economic activity also continues indefinitely at an equilibrium level with a single injection of endogenous money: additional money is not needed to sustain economic activity at a constant level. This contradicts Graziani's assertion that additional money would be needed if workers retained positive bank balances (Graziani 2003: 31), but confirms Keynes's intuition that a "revolving fund of a more or less constant amount" can finance sustained economic activity (Keynes 1937b: 248).

The size of the initial loan L can also be related to the equilibrium value of wages generated by the loan:

$$L = W_E\cdot \frac{((1-s)\cdot P + \omega - r_D)\cdot(\beta - r_D)}{(1-s)\cdot P\cdot(\omega - r_D)\cdot(\beta - r_L)} = 100 \quad (0.6)$$

Two more issues remain to be considered: the impact of debt repayment, and the modelling of growth.

Debt repayment and bank reserves

According to Graziani—and almost all theorists in endogenous money—the repayment of debt destroys the money that was created with it (Graziani 2003: 29-30). I consider this by adding an additional term R_L to represent the repayment of debt. If we relate this to the level of outstanding debt[5], then the amount $R_L.F_L$ is deducted from the firms' only source of money, F_D. Yet to where does it go?

Here Graziani's anti-seignorage condition comes into play: "the use of money must be so regulated as to give no privilege of seigniorage to any agent" (Graziani 2003: 60). This repayment therefore cannot be made to the existing bankers' deposit account B_D, since banks use this account to finance spending on commodities. It must therefore go to a separate, capital account: the banks' reserve account, which I call B_R.

Reserves, once created by the repayment of loans, will be re-lent. This amount will be deducted from the banks' reserve account and deposited in the firms' deposit account—and a matching entry will be made in the firms loan record of account. The complete relations are shown in Table 8.

Bank Assets & Liabilities					
Flows	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Income**
Repayment of debt	$-R_L.F_L$	$-R_L.F_L$			$-R_L.F_L$
Relending of reserves	$+L_R.B_R$	$+L_R.B_R$			$+L_R.B_R$

Bank Reserves		
Time	**Reserve Account**	**Capital**
Repayment of debt	$R_L.F_L$	$+R_L.F_L$
Relending of reserves	$-L_R.B_R$	$-L_R.B_R$
SAM	**Sum**	**0**

Table 8: Repayment and relending

The repayment of loans therefore does not "destroy" money, but transfers it out of income accounts—where it can be used for expenditure—to a reserve account. The proposition that money is destroyed when loans are repaid in part reflects economic conventions that money is the sum of active bank balances. If money is defined that way, then it is indeed destroyed; but I feel that the dynamics of endogenous money creation are more clearly illuminated if we define money in the fundamental Circuitist sense as a token whose transfer settles all commitments between trading parties. That token can then reside in active accounts (deposits) or inactive accounts (reserves). Repayment of loans alters the balance between active and inactive accounts, and thus alters the amount of money in circulation, but it does not destroy the token itself.

Once there, it is an unemcumbered asset of the banks which can then be re-lent—though not spent directly on commodities or services. This adds an important additional insight to the concept of endogenous money: not only do "loans create deposits", but "the repayment of loans creates reserves".

This results in the model shown in equation (0.7):

$$
\begin{aligned}
\tfrac{d}{dt}F_L &= +L_R \cdot B_R - R_L \cdot F_L \\
\tfrac{d}{dt}F_D &= \left(r_D F_D - r_L F_L\right) - (1-s)\cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) \\
\tfrac{d}{dt}B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt}W_D &= (1-s)\cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
\tfrac{d}{dt}B_R &= +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
$$
(0.7)

The simulation results for this model are shown in Figure 3 (with a shorter time span to show the initial dynamics). The new parameters R_L and L_R were given the values of 2 and 3 respectively.

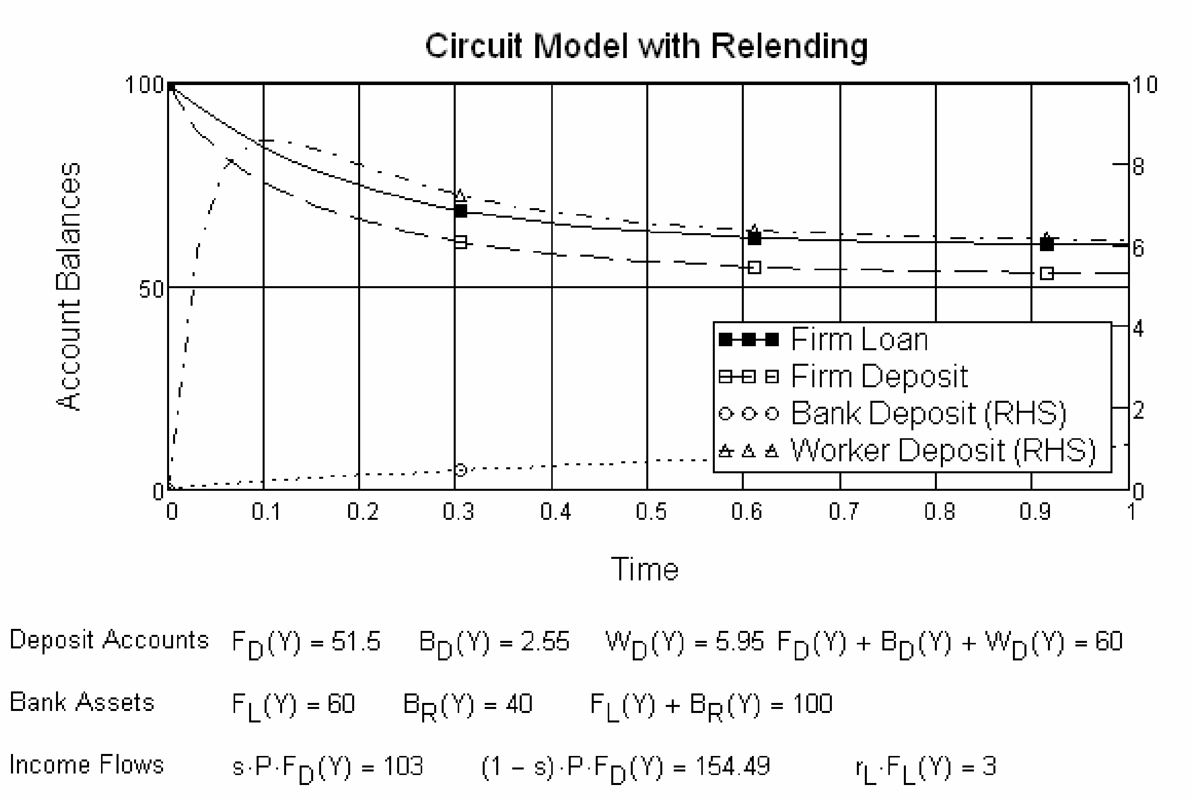


Figure 3: Model with repayment and relending

The equilibrium values are shown in Equation (0.8):

$$\begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \\ B_{R_E} \end{bmatrix} = \frac{1}{L_R + R_L} \cdot \begin{bmatrix} L_R \cdot L \\ L_R \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ L_R \cdot \dfrac{L \cdot (r_L - r_D)}{\beta - r_D} \\ L_R \cdot \dfrac{L \cdot (1-s) \cdot P \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ R_L \cdot L \end{bmatrix} = \begin{bmatrix} 60 \\ 51.5 \\ 2.55 \\ 5.95 \\ 40 \end{bmatrix} \qquad (0.8)$$

It is obvious that money is not destroyed, but turned into reserves that are then available for relending. However, there is a reduction in money in circulation at any one time, equivalent to the proportion of debt that has been repaid. Given the parameters used in this simulation, the amount of circulating money is reduced from 100 to 60 units.

It is thus not money that is "destroyed" by the repayment of debt, but deposits in income accounts. This in turn reduces the amount available for the financing of production, reducing all incomes—including that of banks. The equilibrium levels of income are now:

$$\begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} 103 \\ 3 \\ 159.49 \end{bmatrix} \qquad (0.9)$$

Growth

At this stage, the model accords with Keynes's verbal analysis of the "revolving fund of finance" without growth. The final problem is how to model endogenous money in a growing economy, when "decisions to invest are (e.g.) increasing" and "the extra finance involved will constitute an additional demand for money." (Keynes 1937b: 248).

Accounting for growth integrates Moore's "Horizontalism" into the Circuitist framework (Moore 1988). As Moore argues, firms negotiate "lines of credit" with banks that enable them to expand the available money, subject to the same sum being added to their outstanding debt. New money is thus created by an addition of an identical sum to the firms' deposit and loan accounts Using F_I (for "Firms' Investment") to signify the rate, and relating this to the level of firms' deposit accounts,[6] this introduces a new term $F_I.F_D$ into the columns for F_L and F_D in the final table. I have included the creation and simultaneous transfer of this new money in the banks' reserve account simply to indicate that the endogenous creation of money by firms depends upon the legal right they have negotiated with banks to expand their borrowings.[7]

Bank Assets & Liabilities					
Flows	Assets	Liabilities			SAM
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)	Income
Investment by firms	$+F_I.F_D$	$+F_I.F_D$			$+F_I.F_D$

Bank Reserves		
Time	Reserves	Capital
Investment by firms	$+F_I.F_D - F_I.F_D$	0
SAM	Sum	$+F_I.F_D$

Table 9: Endogenous creation of new money

There is no offsetting transfer between income and capital accounts in this case, so that the term $F_I.F_D$ causes a net increase in the money stock: it is an endogenous source of growth. As a result, rather than having a zero sum, the complete SAM has a positive sum, equal to the amount of new money $F_I.F_D$ being created each year. The overall model, as shown in Equation (0.10), is therefore "dissipative"—in the language of modern dynamic analysis—rather than "conservative", which has important implications for the feasible behaviour of any complete model built on this skeleton.

$$
\begin{aligned}
\tfrac{d}{dt}F_L &= +L_R \cdot B_R - R_L \cdot F_L + F_I \cdot F_D \\
\tfrac{d}{dt}F_D &= \left(r_D F_D - r_L F_L\right) - \left(1-s\right)\cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) + F_I \cdot F_D \\
\tfrac{d}{dt}B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt}W_D &= \left(1-s\right)\cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
\tfrac{d}{dt}B_R &= +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
\tag{0.10}
$$

Though the amount of money and debt in this final model grow exponentially over time, the same relations hold between debt and income deposits, while the overall money stock includes both the sum of deposit accounts and the amount in banks' reserves. At the end of the simulation period (30 years), the endogenous money stock has grown from 100 to 379.13, 228.78 of which is in circulation between firm, bank and worker income accounts, and 150.35 of which is in the banks' reserve account.

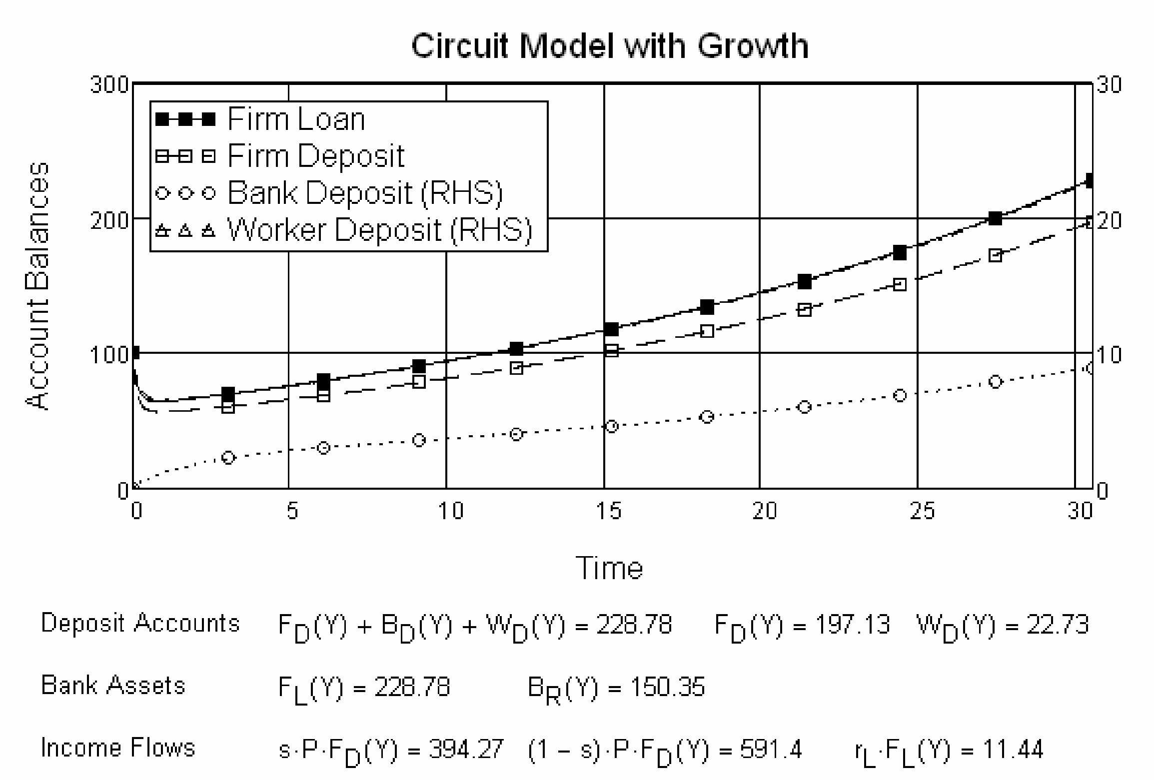


Figure 4: Model with growth

From parameters to behaviours

Like a biological skeleton, this model is designed to have muscles attached, in that its fixed parameters can be replaced by nonlinear behavioral relations that mimic those of real economies. Two that deserve special mention are R_L and F_L,, representing respectively the rate of relending by banks and the rate of new money creation driven by firms.

The latter provides the "Horizontalist" aspect of this skeletal model, and in a general model would be a nonlinear function of firms' expectations of profits (see Keen 1995). The former reflects the Structuralist emphasis on the active role of banks in the credit system. In a financial crisis, this would tend towards zero, while during a period of euphoric expectations the rate of relending would accelerate.

This illustrates another advantage of dynamic modelling over the conventional diagrammatic and static methods that Post Keynesian and Circuitist economists have in the past applied. Diagrammatic methods are necessarily "two dimensional", while static methods make it difficult, if not impossible, to examine causal relations—even when they are correctly specified, which is rarely if ever the case. On the other hand, this properly specified dynamic model enables the integration of the Horizontalist and Structuralist approaches (which could be further embellished by making the spread between r_L and r_D a variable).

Conclusion

Keynes was correct that a “revolving fund of finance” can initiate an indefinite stream of production, and that this fund is a necessary prelude to production itself in a monetary economy. The Circuitist formalisation of the concept of credit money plays an essential role in converting Keynes’s vision from a verbal to a dynamic model, but at the same time, some prevalent Circuitist concepts must be abandoned in favour of Keynes’s accurate insights from 1937.

Both Keynes and Circuitists gain from this model. Keynes is shown, once again, to have correctly identified the dynamics of a monetary production economy, even though he did lacked the assistance of mathematical logic to clarify his argument. Circuitists gain an effective expression of their model, and lose only erroneous conclusions that shackled their capacity to achieve their real goal, of specifying the behaviour of endogenous money in a monetary production economy.

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[1] Similar conclusions are reached in numerous other Circuitist papers from Graziani 1989 on. Rochon puts the problem well: “The existence of monetary profits at the macroeconomic level has always been a conundrum for theoreticians of the monetary circuit… not only are firms unable to create profits, they also cannot raise sufficient funds to cover the payment of interest. In other words, how can *M* become *M*`?” (Rochon 2005: 125).

[2] The Central Bank properly enters the Circuitist model when the banking sector is expanded, so that a seller can deposit the proceeds of a sale in a different bank to that of the buyer. This necessitates a clearing house between banks, which is the primary role of a Central Bank in the Circuitist model. In this paper, for the sake of simplicity, I omit inter-bank dynamics.

[3] Later I apply Graziani’s position that “the demand for bank credit coming from producers depends only on the wage rate and on the number of workers that firms intend to hire” (29) to calculate the size of the initial loan L as a function of the equilibrium wage bill

[4] Again, in a more complete model, each of these stages of the process would have their own equation with its own dynamics; here, for reasons of simplicity and exposition, they are all collapsed into the values of s and P.

[5]It could equally be related to the level of F_D.

[6] It could as easily be related to the level of outstanding loans, and would doubtless have a more complex causal link in a full dynamic model.

[7] In a full model, this could be given a rationing ceiling; however I believe that a better way to indicate banks’ “structuralist” control over lending is to replace R_L with a variable dependent upon financial conditions.